

# C.U.SHAH UNIVERSITY

## Summer Examination-2019

Subject Name : Mathematics-I  
 Subject Code : 4SC01MAT1 Branch : B.Sc. (All)  
 Semester : 1 Date : 16/03/2019 Time : 2:30 To 5:30 Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions: (14)**

- a) What is order of differential equation :  $(y'')^5 + y''' + 2xy = 0$  (1)
- b) The solution of differential equation  $y'' + 2y = 0$  is ..... (1)
  - (a)  $\sin x$  (b)  $\cos 2x$  (c)  $2\sin x$  (d)  $2\cos x$
- c) True/false : Machlaurin's series is particular case of taylor's series. (1)
- d) Write the equation of circle having centre (0 ,0) and radius 5 in polar form. (1)
- e) Find 5<sup>th</sup> derivative of  $\log(2x+7)$ . (1)
- f) True/false: Every system of linear simultaneous homogenous equation is consistent. (1)
- g) Define: Order and degree of differential equation . (2)
- h) Explain the difference between order and rank of matrix. (2)
- i) Give an example of differential equation which is exact also justify it. (2)
- j) Find order and degree of the differential equation (2)

$$\left(\frac{d^8y}{dx^8}\right)^5 + \left(\frac{d^7y}{dx^7}\right)^2 + 2xy = 0.$$

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

- a) Find rank of matrix: (5)
 
$$\begin{bmatrix} 3 & 4 & -2 & 1 & 2 \\ 7 & -3 & 1 & -2 & -1 \\ 3 & 1 & 0 & 1 & 3 \end{bmatrix}$$
- b) Solve  $7x - 3y + 2z = 11, 4x - 6y - 2z = 15, 7x + 3y - 4z = 1$  using Cremer's method. (5)
- c) Find Eigen value of (4)
 
$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 2 & 7 \\ 7 & 7 & 1 \end{bmatrix}$$

**Q-3 Attempt all questions (14)**



- a) Discuss the consistency of the system of equation (5)

$$2x - 3y + 5z = 11, 3x - 4y + 7z = -12, 4x + 10y + 17z = 5.$$

If it is consistent then find its solution.

- b) Find characteristic equation of matrix (5)

$$\begin{bmatrix} 3 & 2 & 3 \\ 0 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

Using it find value of  $A^8 - 6A^7 + 5A^6 - 3A^5 + 5A^4 - 8A^3 - 2A + I$ .

- c) If  $A = \begin{bmatrix} 30 & 2 \\ 10 & 4 \end{bmatrix}$  then verify Cayley Hamilton's theorem. (4)

**Q-4 Attempt all questions (14)**

- a) Solve:  $(x^2 + y^2)dx - 2y^2 dy = 0$ . (5)

- b) Solve:  $\frac{dx}{dy} + \frac{4y}{y^2+1} x = \frac{1}{(y^2+1)^3}$  (5)

- c) Solve:  $x \cos y dx - \frac{x^2}{2} \sin y dy = 0$  (4)

**Q-5 Attempt all questions (14)**

- a) Find equation of sphere which passes through  $(0,0,0)$ ,  $(3,0,0)$ ,  $(0,4,0)$  and  $(0,0,6)$ . (6)

- b) State and prove Leibnitz's theorem for  $n^{\text{th}}$  derivative of product and find  $n^{\text{th}}$  derivative of  $x \sin x$ . (8)

**Q-6 Attempt all questions (14)**

- a) Find  $n^{\text{th}}$  derivative of the following : (6)

(a)  $\frac{1}{(2x+1)(2x+2)}$  (b)  $\frac{2x+1}{x^2-1}$

- b) State  $n^{\text{th}}$  derivative of  $\sin ax$  and  $a^x$  and prove it. (4)

- c) If  $y = e^{ax} \cos(bx+c)$  then show that  $y_n = (a^2 + b^2)^{\frac{n}{2}} e^{ax} \cos\left(bx+c+n\text{th an}^{-1}\left(\frac{b}{a}\right)\right)$  (4)

**Q-7 Attempt all questions (14)**

- a) Express  $x^5 + 4x^4 + 6x^3 - 4x + 1$  as powers of  $x - 2$ . (5)

- b) Find machlurin's series of  $\sinh x + \cosh x$ . (5)

- c) Express  $\sin x \cdot \cos x$  in powers of  $x$  upto  $x^6$ . (4)

**Q-8 Attempt all questions (14)**



a) Evaluate the following : (6)

(1)  $\lim_{x \rightarrow \infty} (e^{x+e^{-x}} - e^x)$ .

(2)  $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$ .

b) Apply Rolle's theorem for  $f(x) = (2x-1)\sin\pi x$  in the interval  $[\frac{1}{2}, 1]$ . (6)

c) Define: Taylor's series . (2)

